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Andrei Zelevinsky, 1953–2013

This volume is dedicated to the memory of Andrei Zelevinsky (30 January 1953 – 10 April 2013), our colleague and our friend.

Andrei was born in Moscow and attended the celebrated mathematical school No. 2, a breeding ground for many famous mathematicians (one of his classmates was Boris Feigin, for example). He graduated from this school in 1969 and entered the Department of Mechanics and Mathematics at Moscow State University. As a member of the USSR International Mathematical Olympiad team—Andrei won a silver medal for the competition—he was directly granted university admission, bypassing cumbersome and not always objective entrance examinations.

It was common for talented students from Moscow State to serve as mentors in leading mathematical high schools. One such mentor in school No. 2 was Joseph Bernstein, who later had great influence on Andrei's mathematics. In early Fall, 1970, Andrei for the first time met Israel M. Gelfand and, as Andrei wrote, this meeting became one of his most life-changing experiences. Andrei attended the Gelfand Seminar, one of the pivotal centers for Moscow mathematics, for almost twenty years. Andrei participated in other seminars, as well, including the seminar run by Alexandre A. Kirillov, his official PhD advisor.

The Gelfand style of doing mathematics: a thorough study of multiple examples, finding hidden and unexpected jewels and then proceeding with a beautiful and deep theory. It was very suitable for Andrei. His own work was rather concrete and very combinatorial. As a true student of Gelfand's school he highly valued the beauty of simplicity in mathematics and, at the same time, was open to fresh abstract ideas.

Andrei navigated the hardships of life in the former USSR with humor and dignity. His reaction to any stupid and undignified act was a sort of surprise: humans just cannot behave like this. But he was also very firm with his principles: it took a lot of civil courage to teach in the early 1980s at underground so-called Jewish People's University

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in Moscow for young students who were denied entry into Moscow State. Several of his students from there later became distinguished mathematicians.

He worked in the mathematical laboratory at the Institute of Earth Science (1977–1985) and the Council for Cybernetics of the Soviet Academy Sciences (1985–1990). In 1990 Andrei and his close collaborator at that time, Mikhail Kapranov, were invited by Cornell University, and in 1991 Andrei accepted an offer from Northeastern University. He and his family lived in Sharon, not far from Boston. Andrei and his wife, Galina, loved Sharon and liked to take their friends for long walks in surrounding parks. His adjustment to American life, including American academia, went surprisingly well. Many of his best results, including the theory of cluster algebras, were obtained at Northeastern.

His behavior after moving to the West was essentially the same: always polite, always frank, and always respectful. He was never jealous of anybody's success and was very supportive to his friends and colleagues. Andrei liked to work with young people, and he payed a lot of attention to their progress.

Among Andrei's distinctions was a Humboldt Research Award (2004) and a University Distinguished Professorship at Northeastern University, the latter having been bestowed upon him posthumously. He gave an invited talk at the International Congress of Mathematicians in 1998. Andrei served on the Scientific Advisory Board of MSRI, and he was a member of the editorial boards of "Advances in Mathematics", "Algebra and Number Theory", "International Mathematical Research Notices", "Journal of Algebraic Combinatorics", "Selecta Mathematica", and "Transformation Groups".

In Fall 2012 Andrei was a key participant in the "Cluster Algebra" program at MSRI, which attracted leading researchers in the field from all over the world. He became ill in December 2012 and was never able to recover. Andrei's mathematical and personal presence will be sorely missed. This volume is a testament to the breadth, depth, and impact of his research and mentoring.

Andrei is survived by his parents Vladlen and Natalia, his wife Galina, his children Leo and Katya, and his grandchildren Gregory and Julia.

Andrei Zelevinsky's mathematical research

The first papers by Andrei were published when he was an undergraduate student. They were devoted to integral geometry and representations over finite fields. A new substantial step in his activity began when Gelfand suggested to him that he work with Joseph Bernstein on p-adic groups, a new subject in Moscow mathematics. As was common in Moscow, they started their studies with a careful analysis of representations of GL_n. Their survey paper was published in 1976; it contained both classical and original results. The development of these ideas appeared in their paper in "Annales scientifiques de l'École normale supérieure", where they introduced the notion of normalized Jacquet functor, and the Geometric Lemma, and proved the irreducibility criterion. In the next paper Andrei gave a complete classification of irreducible representations of GL_n in terms of cuspidal representations. This description is now called the Zelevinsky classification.

By working on this theory Andrei realized that the representation theory of the symmetric group S_n, and of finite linear groups $GL_n(\mathbf{F}_q)$ can be elegantly described using the language of Hopf algebras. In particular, as the integer basis for the Hopf algebra of symmetric functions, the Schur functions have two special properties: first, they have the same structure constant for their multiplication as for their comultiplication, and second, these structure constants are all nonnegative integers. Andrei isolated these two properties as crucial by defining a positive self-dual algebra (PSH for short) and developing a beautiful theory of such algebras. The complete classification of irreducible representations of groups $GL_n(\mathbf{F}_q)$ was obtained by Andrei in a joint paper with Tony Springer.

In the mid-1980s Gelfand proclaimed at his seminar that the next century will be a century of analysis and combinatorics. It was a beginning of a new theory of higher-dimensional hypergeometric functions related to integral geometry which was considered as a natural part of representation theory. Andrei was a central figure in the group of Gelfand's collaborators. The definition of A-hypergeometric system of differential equations given by Andrei, Gelfand, and Mark Graev and later studied by Andrei, Gelfand, and Kapranov appeared as the crown of the new theory. These systems are now known in literature as GKZ systems. Their solutions in terms of power series and integrals led to new problems in hyperplane arrangements, convex polytopes, and matroids. GKZ systems have become household objects in the theory of Calabi–Yau varieties and mirror symmetry.

Attempts to understand relations between the theory of GKZ systems and toric varieties led Andrei, Gelfand, and Kapranov to hyperdeterminants—that is, determinants of multidimensional matrices. They started by rediscovering Cayley's determinant of $2 \times 2 \times 2$ matrices but quickly moved forward to discriminants of polynomials in many variables. Their well known book on this subject is an excellent example of constructing a bridge from old and new algebraic geometry to the theory of convex polytopes.

Also starting in mid-1980s, Andrei became deeply interested in linear bases of representation spaces. In his early papers with Gelfand and Vladimir Retakh, such "good" bases in the coordinate algebra of the base affine space were identified for groups GL_3 and Sp_4. A deep generalization of this program was carried out by Andrei and his student Arkady Berenstein in a series of papers where they developed a technique using convex polyhedral cones. The complete solution was obtained by Andrei and Berenstein in 2001. These bases are closely related to the canonical bases of Lusztig and the crystal bases of Kashiwara.

Andrei's work on "good" bases confirmed deep connections between geometry of Lie groups and their representations. This led Andrei, together with Berenstein and Sergey Fomin, to the study of factorization problems for classical Lie groups, which open a path to the discovery of cluster algebras.

In 2002 Andrei and Fomin published their definition of cluster algebra, one of the most influential discoveries in mathematics during the last two decades, with impacts in algebra, combinatorics, geometry, mathematical physics, low-dimensional topology, and beyond. They understood that many connections between total positivity in Lie groups and dual canonical bases can be explained in the language of simple combinatorial transformations called mutations. They isolated the key role of the Laurent phenomenon for compositions of such mutations. A generalization to the quantum case was then given by Andrei and Berenstein.

Today there are hundreds of papers investigating the structural theory of cluster algebras and their applications in various areas of mathematics and theoretical physics, including integrable systems, statistical physics, Teichmüller theory, and Poisson and symplectic geometry. In addition, Andrei with Harm Derksen and Jerzy Weyman developed a deep theory of mutations of quivers with potentials. Several papers in this volume dedicated to the memory of Andrei demonstrate the central role of cluster algebras in modern mathematics.

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